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Quark production in high-energy proton–nucleus collisions

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Abstract. In this note, we discuss the problem of quark–antiquark pair production in the framework of the color glass condensate. The cross-section can be calculated in closed form for the case of proton–nucleus collisions, where the proton can be considered to be a dilute object. We find that $k_\perp$-factorization is broken by rescattering effects.

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1 Introduction

The color glass condensate (CGC) [1–3] provides a framework for studying hadronic collisions at high energy. Following the ideas developed in the McLerran–Venugopalan model [4–7], it divides the partonic degrees of freedom into hard color sources ρ – partons with a large momentum fraction x that travel at almost the speed of light and that can be considered as frozen over the typical interaction times – and classical color fields A^μ. The latter represent the small x modes, which have a large occupation number and thus can be described classically. The two types of modes are related by the fact that the classical fields obey the Yang–Mills equations, with a source term given by the hard color sources.

The hard sources are random variables (the partons at large x come in a different configuration in each collision), whose statistical distribution is described by a functional density $W_{x_0}[\rho]$. The subscript x_0 in this functional is the separation between the degrees of freedom that are described as sources and those that are described as fields. Upon changing this boundary, the functional W_{x_0} must change according to a renormalization group equation, known as the JIMWLK equation [8–19]. The initial condition for this evolution equation is in principle nonperturbative. However, in the case of a large projectile like a nucleus, it has been argued by McLerran and Venugopalan that a good model for this initial condition is given by a Gaussian [4–7]. Such a Gaussian should remain a reasonable description of the hard sources in a nucleus as long as x_0 does not become too small.

In a collision between two hadronic projectiles, each projectile is described by its own color source $\rho_{1,2}$, which correspond to partons moving at the speed of light in opposite directions. The first step in studying this collision process is to solve the Yang–Mills equations in the presence of these two source terms. So far, this has been achieved analytically only in the case where at least one of the sources is assumed to be a weak source and treated perturbatively (i.e. the solution of Yang–Mills equations is expanded order by order in powers of this weak source) [20, 21]. This situation of course prevails in the collision between two small, non-saturated, projectiles, like two protons at not too small x. It is also relevant in the case of a small and a large projectile, like proton–nucleus collisions, or deuteron–nucleus collisions as performed at RHIC. The solution of Yang–Mills equations in this asymmetrical situation will be reviewed in Sect. 2. The case of nucleus–nucleus collisions, where none of the color sources can be considered to be weak, is at the moment out of reach of analytic calculations, but can be solved numerically [22–26].

Once the classical gauge fields are known, physical cross-sections can be calculated. It is straightforward to obtain the gluon multiplicity[1] at the classical level, since it is simply a matter of doing a Fourier decomposition of the gauge fields. In order to obtain the multiplicity of the produced quark–antiquark pairs, one needs the retarded propagator of a quark in the presence of the previously de-

[1] In the case where both sources are strong, and thus the classical gauge fields are also strong fields, it is much simpler to evaluate inclusive quantities like multiplicities instead of cross-sections for more exclusive processes. Indeed, the latter involves the calculation of time-ordered amplitudes in the presence of the classical gauge field. The fact that this gauge field is time dependent in a collision process renders this problem very difficult in general due to the presence of vacuum-to-vacuum diagrams [27]. On the contrary, the calculation of multiplicities only involves retarded amplitudes in the classical field, for which there are no vacuum-to-vacuum diagrams.

termined classical gauge field. Again, in situations where the background gauge field is only known numerically, the quark propagator can only be computed numerically as well [28–30]. For collisions between a small and a large projectile however, one can obtain the quark production cross-section analytically, as was already the case for the classical color fields. The result of this calculation will be presented in Sect. 3.

Finally, as a last step involved in calculating an observable relevant for hadronic collisions, one must perform the average over the configurations of the hard color sources, weighted by a factor $W[\rho]$ for each projectile. This average cannot be performed analytically in general, except in some special cases like when $W[\rho]$ is a Gaussian functional. The source averages that we need in the expression of the quark production cross-section will be given in Sect. 4.

At this point, the quark production cross-section is expressed as a multi-dimensional integral that cannot be further simplified analytically. One observes that $k_\perp$-factorization is broken by rescattering effects, contrary to what happened in the simpler case of gluon production. Some implications of this formula are discussed for single quark production and for quarkonium production in Sect. 5.

2 Gauge fields

When studying the collision of two hadrons in the framework of the color glass condensate, the first step is to determine the classical color field created in this collision[2]. One has to solve the Yang–Mills equations,

$$[D_\mu, F^{\mu\nu}] = J^\nu \; . \tag{1}$$

For these equations to be compatible with the constraints provided by the Jacobi identity, the color current J^ν that appears in the RHS must be covariantly conserved:

$$[D_\nu, J^\nu] = 0 \; . \tag{2}$$

This condition implies that the current in general receives corrections to all orders in the color sources. At lowest order in the sources, it is given by

$$J^\nu(x) = g\delta^{\nu+}\delta(x^-)\rho_1(\boldsymbol{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_2(\boldsymbol{x}_\perp) \; , \tag{3}$$

where ρ_1 and ρ_2 are the distributions of color charge in the hadron moving to the right and to the left respectively. The previous three equations must be supplemented by a gauge condition. In our work, we have used the Lorenz-covariant gauge

$$\partial_\mu A^\mu = 0 \; . \tag{4}$$

Thus far, the solution of this set of equations is not known to all orders in both ρ_1 and ρ_2 (although numerical solutions have been obtained). The best one can achieve is to obtain the solution to first order in ρ_1 (the source describing the proton) and all orders in ρ_2 (the source describing the nucleus). In the Lorenz gauge, this solution has a nice and compact form, which reads, in Fourier space [21],

$$\begin{aligned} A^\mu(q) &= A^\mu_p(q) \\ &+ \frac{\mathrm{i}g}{q^2 + \mathrm{i}q^+\epsilon} \int \frac{\mathrm{d}^2\boldsymbol{k}_{1\perp}}{(2\pi)^2} \\ &\times \Big\{ C^\mu_U(q, \boldsymbol{k}_{1\perp}) \left[U(\boldsymbol{k}_{2\perp}) - (2\pi)^2 \delta(\boldsymbol{k}_{2\perp}) \right] \\ &\quad + C^\mu_V(q) \left[V(\boldsymbol{k}_{2\perp}) - (2\pi)^2 \delta(\boldsymbol{k}_{2\perp}) \right] \Big\} \frac{\rho_1(\boldsymbol{k}_{1\perp})}{k^2_{1\perp}} \; . \end{aligned} \tag{5}$$

In this formula, the first term is the color field of the proton alone:

$$A^\mu_p(q) = 2\pi g \delta^{\mu+} \delta(q^-) \frac{\rho_1(\boldsymbol{q}_\perp)}{q^2_\perp} \; . \tag{6}$$

In the second term, $\boldsymbol{k}_{1\perp}$ is the momentum coming from the proton and $\boldsymbol{k}_{2\perp}$, defined as $\boldsymbol{k}_{2\perp} \equiv \boldsymbol{q}_\perp - \boldsymbol{k}_{1\perp}$, is the momentum coming from the nucleus. The 4-vectors[3] C^μ_U and C_V have been introduced in [21], and U, V are the Fourier transforms of Wilson lines in the adjoint representation of $SU(N)$:

$$\begin{aligned} U(\boldsymbol{x}_\perp) &\equiv \mathcal{P}_+ \exp\left[\mathrm{i}g \int_{-\infty}^{+\infty} \mathrm{d}z^+ A^-_A(z^+, \boldsymbol{x}_\perp) \cdot T \right] \; , \\ V(\boldsymbol{x}_\perp) &\equiv \mathcal{P}_+ \exp\left[\mathrm{i}\frac{g}{2} \int_{-\infty}^{+\infty} \mathrm{d}z^+ A^-_A(z^+, \boldsymbol{x}_\perp) \cdot T \right] \; , \end{aligned} \tag{7}$$

where A^-_A is the gauge field of the nucleus alone. The T^a are the generators of the adjoint representation of $SU(N)$ and $\mathcal{P}_+$ denotes a "time ordering" along the z^+ axis. The peculiarity of this result is that it contains a Wilson line (V) with an unusual factor $1/2$ in the exponent. Such a Wilson line must be an artifact of the gauge we use, and its cancellation from physical quantities is a non-trivial test of the gauge invariance of the final result[4].

3 Quark production

3.1 Pair production amplitude

Once the gauge fields have been obtained, the probability of producing a quark–antiquark pair in the collision is

[2] An exception to this is the case where one treats one of the two hadrons using the standard collinear factorization instead of a description based on the CGC [31–36]. In this case, one needs only the gauge field produced by the other hadron alone (which is of course much simpler).

[3] They are related to Lipatov's effective vertex C^μ_{L} by $C^\mu_U + \frac{1}{2}C^\mu_V = C^\mu_{\mathrm{L}}$.

[4] The gauge field produced in pA collisions has also been determined in the Fock–Schwinger gauge $x^+A^- + x^-A^+ = 0$ [20] and in the gauge $A^+ = 0$ [37]. It has been checked explicitly that although the gauge fields are not the same in these other gauges, they all lead to the very same expression for the multiplicity of produced gluons.

given by[5]

$$\omega_{\boldsymbol{q}} \frac{\mathrm{d}P_{Q\overline{Q}}}{\mathrm{d}^3\boldsymbol{q}} = \frac{1}{16\pi^3} \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3 2\omega_{\boldsymbol{p}}} \left|\overline{u}(\boldsymbol{q})T(q,-p)v(\boldsymbol{p})\right|^2 \;, \quad (8)$$

where $\omega_{\boldsymbol{q}} \equiv \sqrt{\boldsymbol{q}^2+m^2}$ is the on-shell energy of a quark, and $T(q,-p)$ the time-ordered propagator (amputated of its external legs) of a quark in the presence of the previously obtained color field, with incoming momentum $-p$ and outgoing momentum q. This probability must be integrated over the impact parameter of the collision in order to obtain the pair production cross-section.

As explained in [38], there is one technical complication when using the gauge field given in (5) in order to calculate the pair production amplitude. Indeed, it turns out that one of the terms in the gauge field is proportional to $\delta(x^+)$ in coordinate space, which means that such a term allows the vertex where the pair is being produced to be located inside the nucleus. Practically, this means that this term must be considered separately, because its contribution can only be calculated by smearing out the thickness of the nucleus. When this is done properly, one obtains a pair production amplitude in which the spurious Wilson line V does not appear any longer:

$$\begin{aligned}\mathcal{M}_F(\boldsymbol{q},\boldsymbol{p}) = g^2 &\int \frac{\mathrm{d}^2\boldsymbol{k}_{1\perp}}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}_{\perp}}{(2\pi)^2}\frac{\rho_{p,a}(\boldsymbol{k}_{1\perp})}{k_{1\perp}^2}\int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \\ &\times \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\mathrm{e}^{\mathrm{i}(\boldsymbol{p}_\perp+\boldsymbol{q}_\perp-\boldsymbol{k}_\perp-\boldsymbol{k}_{1\perp})\cdot\boldsymbol{y}_\perp} \\ &\times \overline{u}(\boldsymbol{q})\{T_{q\bar{q}}(\boldsymbol{k}_{1\perp},\boldsymbol{k}_\perp)[\widetilde{U}(\boldsymbol{x}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{y}_\perp)] \\ &\qquad + T_{\mathrm{g}}(\boldsymbol{k}_{1\perp})[t^b U^{ba}(\boldsymbol{x}_\perp)]\}v(\boldsymbol{p}) \;, \end{aligned} \quad (9)$$

where we denote

$$\begin{aligned} &T_{q\bar{q}}(\boldsymbol{k}_{1\perp},\boldsymbol{k}_\perp) \equiv \\ &\quad \frac{\gamma^+(\not{q}-\not{k}+m)\gamma^-(\not{q}-\not{k}-\not{k}_1+m)\gamma^+}{2p^+[(\boldsymbol{q}_\perp-\boldsymbol{k}_\perp)^2+m^2]+2q^+[(\boldsymbol{q}_\perp-\boldsymbol{k}_\perp-\boldsymbol{k}_{1\perp})^2+m^2]}, \\ &T_{\mathrm{g}}(\boldsymbol{k}_{1\perp}) \equiv \frac{\not{C}_{\mathrm{L}}(p+q,\boldsymbol{k}_{1\perp})}{(p+q)^2} \;. \end{aligned} \quad (10)$$

In (9), $\widetilde{U}$ is the same Wilson line as U, except that it is now in the fundamental representation of the gauge group.

3.2 Pair cross-section

At this point, it is a simple matter of squaring this amplitude, performing the average over the color sources in both projectiles, and integrating over the impact parameter of the collision in order to obtain the cross-section for pair production. This gives

$$\begin{aligned} &\frac{\mathrm{d}\sigma_{Q\overline{Q}}}{\mathrm{d}^2\boldsymbol{q}_\perp \mathrm{d}y_q \mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}y_p} \\ &= \frac{\alpha_s^2 N}{8\pi^4(N^2-1)} \int_{\boldsymbol{k}_{1\perp},\boldsymbol{k}_{2\perp}} \frac{\delta(\boldsymbol{p}_\perp+\boldsymbol{q}_\perp-\boldsymbol{k}_{1\perp}-\boldsymbol{k}_{2\perp})}{\boldsymbol{k}_{1\perp}^2\boldsymbol{k}_{2\perp}^2} \\ &\times \Big\{ \int_{\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\not{p}-m)T_{q\bar{q}}^{*\prime}\Big]\phi_A^{q\bar{q},q\bar{q}}(\boldsymbol{k}_{2\perp};\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp) \\ &+ \int_{\boldsymbol{k}_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\not{p}-m)T_{\mathrm{g}}^{*}\Big]\phi_A^{q\bar{q},g}(\boldsymbol{k}_{2\perp};\boldsymbol{k}_\perp) + \mathrm{h.c.} \\ &+ \mathrm{tr}\Big[(\not{q}+m)T_{\mathrm{g}}(\not{p}-m)T_{\mathrm{g}}^{*}\Big]\phi_A^{g,g}(\boldsymbol{k}_{2\perp})\Big\}\,\varphi_p(\boldsymbol{k}_{1\perp}) \;, \end{aligned} \quad (11)$$

The function φ_p, and the various ϕ_A's are correlators of color sources, defined as follows [38]:

$$\begin{aligned} \varphi_p(\boldsymbol{l}_\perp) &\equiv \frac{\pi^2 R_p^2 g^2}{l_\perp^2}\int_{\boldsymbol{x}_\perp} \mathrm{e}^{\mathrm{i}\boldsymbol{l}_\perp\cdot\boldsymbol{x}_\perp}\left\langle\rho_{1,a}(0)\rho_{1,a}(\boldsymbol{x}_\perp)\right\rangle \;, \\ \phi_A^{g,g}(\boldsymbol{l}_\perp) &\equiv \frac{\pi^2 R_A^2 l_\perp^2}{g^2 N}\int_{\boldsymbol{x}_\perp} \mathrm{e}^{\mathrm{i}\boldsymbol{l}_\perp\cdot\boldsymbol{x}_\perp}\left\langle U(0)U^\dagger(\boldsymbol{x}_\perp)\right\rangle_{aa} \;, \\ \phi_A^{q\bar{q},g}(\boldsymbol{l}_\perp;\boldsymbol{k}_\perp) &\equiv \frac{2\pi^2 R_A^2 l_\perp^2}{g^2 N}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} \mathrm{e}^{\mathrm{i}(\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp+(\boldsymbol{l}_\perp-\boldsymbol{k}_\perp)\cdot\boldsymbol{y}_\perp)} \\ &\qquad \times \mathrm{tr}\left\langle\widetilde{U}(\boldsymbol{x}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{y}_\perp)t^b U_{ba}(0)\right\rangle \;, \\ \phi_A^{q\bar{q},q\bar{q}}(\boldsymbol{l}_\perp;\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp) &\equiv \frac{2\pi l_\perp^2}{g^2 N} \\ &\times \int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{u}_\perp,\boldsymbol{v}_\perp} \mathrm{e}^{\mathrm{i}(\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp-\boldsymbol{k}'_\perp\cdot\boldsymbol{u}_\perp)}\mathrm{e}^{\mathrm{i}(\boldsymbol{l}_\perp-\boldsymbol{k}_\perp)\cdot\boldsymbol{y}_\perp}\mathrm{e}^{-\mathrm{i}(\boldsymbol{l}_\perp-\boldsymbol{k}'_\perp)\cdot\boldsymbol{v}_\perp} \\ &\times \mathrm{tr}\left\langle\widetilde{U}(\boldsymbol{x}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{y}_\perp)\widetilde{U}(\boldsymbol{v}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{u}_\perp)\right\rangle \;. \end{aligned} \quad (12)$$

It is not possible to write this cross-section in terms of a single distribution describing the nucleus, and therefore a strict $k_\perp$-factorization is impossible to achieve[6]. However, a relaxed form of $k_\perp$-factorization remains true, termed "non-linear $k_\perp$-factorization" in [41,42], provided one introduces three distinct functions describing the content of nucleus. The three ϕ_A's are not entirely independent, since they are related by some sum rules [38], but they nevertheless probe different correlations among the color sources contained in the nucleus.

3.3 Single quark cross-section

It is also possible to integrate out the phase-space of the antiquark in order to obtain the inclusive single quark

[5] As shown in [27], this formula is in general incomplete because it does not include the contribution of vacuum-to-vacuum diagrams, which is in principle required by unitarity. However, in our approximation where only the leading order in the source ρ_1 is kept, this correction can be safely neglected. Note also that in this approximation, it would be equivalent to use the retarded quark propagator, instead of the time-ordered propagator.

[6] Let us recall that in the case of gluon production, the cross-section can be expressed in terms of the function $\phi_A^{g,g}$ only [40, 20,21]. Therefore, even though this function is not the canonical unintegrated gluon distribution of the nucleus, one has a cross-section which is exactly $k_\perp$-factorizable.

cross-section[7]. One obtains [21]

$$\frac{\mathrm{d}\sigma_Q}{\mathrm{d}^2\boldsymbol{q}_\perp dy_q} = \frac{\alpha_s^2 N}{8\pi^4(N^2-1)}\int\frac{\mathrm{d}p^+}{p^+}\int_{\boldsymbol{k}_{1\perp},\boldsymbol{k}_{2\perp}}\frac{1}{\boldsymbol{k}_{1\perp}^2\boldsymbol{k}_{2\perp}^2}$$
$$\times\Big\{\mathrm{tr}\Big[(\not q+m)T_{q\bar{q}}(\not p-m)T_{q\bar{q}}^*\Big]\frac{C_F}{N}\phi_A^{q,q}(\boldsymbol{k}_{2\perp})$$
$$+\int_{\boldsymbol{k}_\perp}\mathrm{tr}\Big[(\not q+m)T_{q\bar{q}}(\not p-m)T_{\mathrm{g}}^*\Big]\phi_A^{q\bar{q},g}(\boldsymbol{k}_{2\perp};\boldsymbol{k}_\perp)+\mathrm{h.c.}$$
$$+\mathrm{tr}\Big[(\not q+m)T_{\mathrm{g}}(\not p-m)T_{\mathrm{g}}^*\Big]\phi_A^{g,g}(\boldsymbol{k}_{2\perp})\Big\}\varphi_p(\boldsymbol{k}_{1\perp}). \quad (13)$$

A new distribution appears in the single quark cross-section, defined as

$$\phi_A^{q,q}(l_\perp) \equiv \frac{2\pi^2 R_A^2 l_\perp^2}{g^2 N}\int_{\boldsymbol{x}_\perp}\mathrm{e}^{\mathrm{i}\boldsymbol{l}_\perp\cdot\boldsymbol{x}_\perp}\mathrm{tr}\left\langle\widetilde{U}(0)\widetilde{U}^\dagger(\boldsymbol{x}_\perp)\right\rangle . \quad (14)$$

Once again, strict $k_\perp$-factorization is broken: one needs three different correlation functions in order to write this cross-section.

4 Correlators of Wilson lines

It is in general extremely difficult to calculate the correlators of Wilson lines that appear in the previous formulae for the pair and single quark cross-sections. In principle, one would have to solve the JIMWLK equation, which is a question that so far has not received a full (numerical or otherwise) answer, despite some encouraging attempts [43].

One can however calculate these correlators analytically in the McLerran-Venugopalan model where the distribution of color sources in the nucleus has the Gaussian form

$$W[\rho] = \exp\left[-\int_{\boldsymbol{x}_\perp}\frac{\rho_a(\boldsymbol{x}_\perp)\rho_a(\boldsymbol{x}_\perp)}{2\mu_A^2}\right] . \quad (15)$$

Since this model is quite relevant at moderate values of x for large nuclei, the closed expressions one obtains from it are of interest in assessing issues such as the magnitude of the breaking of $k_\perp$-factorization, or in order to make phenomenological predictions in this kinematical domain. The expressions of the various correlators have been derived in [38]. Let us denote

$$\Gamma(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp) \equiv g^4\int_{\boldsymbol{z}_\perp}\left[G_0(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)-G_0(\boldsymbol{y}_\perp-\boldsymbol{z}_\perp)\right]^2 , \quad (16)$$

where G_0 is the free 2-dimensional propagator

$$G_0(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp) \equiv \int\frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^2}\frac{\mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)}}{\boldsymbol{k}_\perp^2} . \quad (17)$$

The simplest ones are the 2-point correlators

$$\left\langle U(0)U^\dagger(\boldsymbol{x}_\perp)\right\rangle_{aa} = (N^2-1)\mathrm{e}^{-\frac{N}{2}\mu_A^2\Gamma(\boldsymbol{x}_\perp)} ,$$

$$\mathrm{tr}\left\langle\widetilde{U}(0)\widetilde{U}^\dagger(\boldsymbol{x}_\perp)\right\rangle = N\mathrm{e}^{-\frac{C_f}{2}\mu_A^2\Gamma(\boldsymbol{x}_\perp)} . \quad (18)$$

Since the 3- and 4-point correlators have quite complicated expressions, we will only quote here the simplified results for the large N limit[8]:

$$\mathrm{tr}\left\langle\widetilde{U}(\boldsymbol{x}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{y}_\perp)t^bU_{ba}(0)\right\rangle = NC_f\mathrm{e}^{-\frac{N}{4}\mu_A^2[\Gamma(\boldsymbol{x}_\perp)+\Gamma(\boldsymbol{y}_\perp)]} ,$$
$$\mathrm{tr}\left\langle\widetilde{U}(\boldsymbol{x}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{y}_\perp)\widetilde{U}(\boldsymbol{v}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{u}_\perp)\right\rangle = NC_f\mathrm{e}^{-\frac{N}{4}\mu_A^2[\Gamma(\boldsymbol{x}_\perp-\boldsymbol{u}_\perp)+\Gamma(\boldsymbol{y}_\perp-\boldsymbol{v}_\perp)]} . \quad (19)$$

One sees that the 3- and 4-point correlators are expressed entirely in terms of 2-point correlators in the large N_c limit. This simplifies the computation enormously.

5 Discussion

5.1 Generalities on $k_\perp$-factorization

The two terms in (9) have a simple interpretation. The first term, containing two Wilson lines in the fundamental representation, corresponds to a process in which the quark–antiquark pair is produced before the collision with the nucleus. The pair then goes through the nucleus and scatters off the color charges present in the nucleus. The second term, which contains only one Wilson line in the adjoint representation, corresponds to a process in which the gluon coming from the proton goes through the nucleus and produces the quark–antiquark pair only after the collision with the nucleus.

Thanks to the identity

$$\widetilde{U}(\boldsymbol{x}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{x}_\perp) = t^aU_{ba}(\boldsymbol{x}_\perp) \quad (20)$$

between Wilson lines in the fundamental and adjoint representations, one sees readily that $k_\perp$-factorization is recovered in the limit where the difference $\boldsymbol{x}_\perp-\boldsymbol{y}_\perp$ between the transverse coordinates of the quark and the antiquark is neglected. In other words, $k_\perp$-factorization should be a valid approximation in any kinematical regime where the typical transverse size of the pair is small. This is expected to be the case if the quark mass is large, or if the transverse momenta in the final state are large.

There is another way to reach the same conclusion from (9). If the dependence of the factor $T_{q\bar{q}}(\boldsymbol{k}_{1\perp},\boldsymbol{k}_\perp)$ on $\boldsymbol{k}_\perp$ is neglected, then the integration over $\boldsymbol{k}_\perp$ in the amplitude produces a $\delta(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)$. This automatically ensures $k_\perp$-factorization. Since the typical $\boldsymbol{k}_\perp$ is of order Q_s, it is possible to neglect $\boldsymbol{k}_\perp$ in the factor $T_{q\bar{q}}$ if there is some other momentum scale (either m or $\boldsymbol{q}_\perp$) much harder than Q_s.

[7] Single quark production has also been studied by Tuchin for Gaussian correlators. Unfortunately, his results are in coordinate space and cannot be easily compared to [39].

[8] It is possible, albeit quite cumbersome, to evaluate the 3-point function numerically without doing the large N approximation [44].

5.2 Quark production

The simplest physical observable to look at is the cross-section for single quark production, obtained by integrating out the phase-space of the anti-quark. This partonic cross-section can be converted into the cross-section for D or B mesons production, by convolution with the appropriate fragmentation function. Since quite a lot of phenomenology has been done in the framework of $k_\perp$-factorization, it is particularly interesting to investigate the importance of the breaking of this factorization, and its dependence on parameters such as the quark mass, the quark transverse momentum, and the saturation scale in the nucleus.

A numerical computation of (13) is under way, and detailed results will be reported elsewhere [44]. One can nevertheless point to the following qualitative trends.

(1) The magnitude of the breaking of $k_\perp$-factorization decreases as the quark mass increases. Indeed, since the terms that break $k_\perp$-factorization correspond to extra rescatterings, it is natural that massive quarks are less sensitive to these effects than light quarks.

(2) The magnitude of the breaking of $k_\perp$-factorization is maximal for a transverse momentum $q_\perp \sim Q_\mathrm{s}$ of the quark, where Q_s is the saturation scale in the nucleus. One recovers $k_\perp$-factorization when the quark transverse momentum becomes much larger than all the other scales.

(3) If Q_s remains smaller or comparable to the quark mass and transverse momentum, the corrections due to the breaking of $k_\perp$-factorization enhance the cross-section. This is interpreted as a threshold effect: having more rescatterings tends to push a few more $Q\overline{Q}$ pairs just above the kinematical production threshold.

(4) If Q_s is large compared to the mass of the quark, then the corrections due to the breaking of $k_\perp$-factorization tend to reduce the cross-section at small transverse momentum. Since the typical momentum transfer in a scattering is of the order of Q_s, it is indeed more difficult to produce light quarks with a small momentum if they scatter more.

In addition, it will be interesting to investigate the effect of this breaking of $k_\perp$-factorization on the Cronin effect and its rapidity dependence, in order to see whether the Cronin effect for heavy quarks follows the same pattern as for gluon production [21,45,46].

5.3 Quarkonium production

Quarkonium production in high-energy heavy ion collisions is a key observable for diagnosing quark gluon plasma formation [47]. The anomalous suppression of the J/ψ, compared with the baseline of the nuclear suppression, was indeed observed at CERN-SPS [48]. In the experimental analysis, the normal dissociation of the J/ψ state by the nucleons is assumed to be independent. At higher energies, however, the nuclei in their center-of-mass frame are Lorentz-contracted to be narrower than the characteristic size of the quarkonium. Thus one expects the coherence between the interactions with the nucleons to be more important. In the rest frame of a nucleus, on the other hand, the internal motion of the heavy quarks is frozen during the interactions.

A model of the quarkonium projected on a target nucleus at high energy is considered in [49], which is formulated as an eikonal propagation of the bound heavy quarks in the random color background fields in the nucleus. The multiple interactions with the color gauge fields result in the random walk of the quarkonium state in the momentum and color spaces. The survival probability of the bound state, the overlap of the initial and final pair state, is shown to be damped like a power-law rather than exponentially in the effective target thickness L. This phenomenon is dubbed “super-penetration” in the QED case.

In order to understand the baseline of the normal nuclear suppression in the high-energy heavy ion collisions, it would be crucial to investigate the nuclear effect on the production process of quarkonium rather than the attenuation of the formed resonance [50,51]. In this context, the analytic expression for the quark production amplitude in the pA collision within the MV model will provide a good starting point for the further study of quarkonium formation. The multiple scattering effect in the nucleus target is appropriately included in the framework, and it will suppress the probability for producing the quark pair with small relative momentum. We speculate that the formation probability of the small-momentum pair will be suppressed in some power of the target size, and not exponentially, reflecting the random walk nature of the multiple scatterings.

6 Conclusions

The calculation of the production quark–antiquark pairs in the framework of the color glass condensate, in the case of proton–nucleus collisions, shows unambiguously that $k_\perp$-factorization is broken. This is to be contrasted to what was observed for gluon production where this factorization is preserved, despite the presence of important rescattering effects in the nucleus. In the case of quark production, one needs more correlators in order to describe the nucleus. In particular, the cross-section depends on correlators of three and four Wilson lines. This opens the possibility of obtaining more detailed information about correlations among color charges, as a function of rapidity. Conversely, experiments can test in detail CGC predictions for these higher point correlators.

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